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Preface to the Special Issue on Riordan Arrays

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Riordan arrays provide a flexible algebraic framework for enumerative combinatorics. A Riordan array is an infinite lower-triangular matrix determined by two formal power series, and its matrix operations translate naturally into multiplication and composition of generating functions. This viewpoint makes it possible to organize, derive, and transform combinatorial identities while connecting the subject with lattice-path enumeration, inverse relations, orthogonal polynomials, special sequences, and other areas of algebraic and enumerative combinatorics.

The modern theory began with the influential paper of Shapiro, Getu, Woan, and Woodson [3], which introduced the Riordan group and established its basic algebraic structure. Sprugnoli's work on combinatorial sums and the Abel–Gould identity [5, 6] demonstrated the effectiveness of Riordan methods for proving and extending classical identities. The theory has since developed in many directions. Barry's *Riordan Arrays: A Primer* [1] made the subject accessible to a broad audience, while the monograph *The Riordan Group and Applications* [4] provides a comprehensive account of its foundations, extensions, and applications. The survey by Davenport, Frankson, Shapiro, and Woodson [2] offers a more recent overview of the subject and several of its principal generalizations.

The continuing growth of the field has been accompanied by a series of international meetings devoted to Riordan arrays and related topics. These meetings have brought together researchers working in infinite matrices, enumerative combinatorics, generating functions, lattice paths, and neighboring areas of discrete mathematics, and they have encouraged collaborations across institutions and countries.

The symposium series began in Seoul, South Korea, in August 2014, as an invited minisymposium of the Nineteenth International Linear Algebra Society Conference. Subsequent meetings were held in Lecco, Italy, in 2015; Bloomington, Illinois, USA, in 2016; Madrid, Spain, in 2017; Busan, South Korea, in 2018; and Sanya, China, in 2019. During the pandemic period, international activity continued in virtual form, including the 2022 Workshop on Combinatorial Species, Operads, Riordan Arrays, and Related Topics hosted by Sungkyunkwan University. The Ninth International Symposium on Riordan Arrays and Related Topics was held at Howard University in Washington, DC, from June 3 to June 5, 2024.

Howard University was a particularly meaningful venue for the ninth symposium. The four authors of the foundational paper introducing the Riordan group—Louis W. Shapiro, Seyoum Getu, Wen-Jin Woan, and Leon C. Woodson—were all affiliated with Howard University's Department of Mathematics when that paper appeared in 1991 [3]. Hosting the meeting at Howard therefore represented both a celebration of current research and a return to one of the intellectual homes of the subject.

As this special issue was being prepared, the Riordan-array community mourned the loss of three important contributors: Seyoum Getu, Renzo Sprugnoli, and Emanuele Munarini. Getu was a coauthor of the foundational Riordan-group paper and a long-serving member of the Howard University mathematics community. Sprugnoli's pioneering work greatly expanded the use of Riordan arrays in combinatorial summation, generating-function methods, and the analysis of algorithms. Munarini's research connected Riordan and Sheffer methods with formal power series, polynomial sequences, posets, graph theory, and a wide range of enumerative structures. Their scholarship, generosity, and service helped shape the field, and their influence will remain visible in its continuing development.

This special issue contains papers presented at the ninth symposium together with closely related contributions. The articles illustrate the breadth and vitality of current research on Riordan arrays, including algebraic properties of the Riordan group, extensions and generalizations of Riordan arrays, lattice-path and tree models, combinatorial identities and bijections, production matrices, and analytic methods involving generating func-

tions. Taken together, they show both the maturity of the theory and the many directions in which it continues to grow.

A particularly meaningful contribution to this volume is Donatella Merlini's memorial tribute to Emanuele Munarini. Her essay records his mathematical range, his longstanding participation in the Riordan-array community, and the kindness and generosity with which he supported colleagues and students. It is fitting that a special issue arising from this conference should preserve a record of his work and his place within the community.

We thank the authors for their contributions, the referees for their careful and thoughtful reports, and the editorial staff of *Enumerative Combinatorics and Applications* for their assistance in preparing this special issue. We hope that the papers collected here will stimulate further research and collaboration in the theory of Riordan arrays and its many applications.

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