Enumerative Combinatorics Applications

אוניברסיטת חיפה
University of Haifa
جامعة حيفا

ICECA

International Conference
Enumerative Combinatorics and Applications
University of Haifa – Virtual – August 25-27, 2025

GENERALIZED HIGHER SPECHT POLYNOMIALS AND THEIR STABILIZATIONS

SHAUL ZEMEL

Einstein Institute of Mathematics, the Hebrew University of Jerusalem, Edmund Safra Campus, Jerusalem 91904, Israel

The Specht polynomials form a basis for the lowest degree realization of the irreducible representation of S_n that is associated with the appropriate shape on polynomials in n variables. The higher Specht polynomials of Ariki, Terasoma, and Yamada produce additional realizations these representations, defined originally in order to decompose the ring R_n of co-invariants. There are more generalized quotients $R_{n,k}$, defined by Haglund, Rhoades, and Shimozono, that admit Specht bases as presented by Gillespie and Rhoades.

We show how these polynomials, and their generalizations, can be normalized in a compatible way such that they produce stable versions, as eventually symmetric functions. This presents explicitly the space of homogeneous polynomials of degree d in n variables as a stable collection of representations. If time permits, we describe how the representations arising from the orbits of the actions producing $R_{n,k}$ also become stable representations by these normalizations.

The stable representations that we get have limits, which are representations of infinite symmetric groups on eventually symmetric functions (also known as almost symmetric functions). If $\tilde{\Lambda}$ is the ring of these functions, then we prove that the direct limit arising in this manner is the maximal completely reducible sub-representation of the part of $\tilde{\Lambda}_d$ that is homogeneous of degree d, and state a conjecture about a filtration on these full representations, with maximal completely reducible quotients.